JHEP HYPER VERSION

Received: *January 31, 2001*, Accepted: *May 09, 2001*
Revised: *May 08, 2001*

The static PQCD potential with modified boundary conditions

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Abstract: We calculate the potential between two static quarks in QCD using modified boundary conditions for the perturbative expansion. Through a change of the Feynman $i\varepsilon$-prescription we effectively add a "sea" of gluons to the asymptotic states with energies below a given scale Λ. We find that the standard result for the static potential gets corrections of order Λ^2/Q^2 both at small and large momentum transfers Q^2. After resummation of the infrared sensitive corrections we find that the running coupling $\alpha_s(Q^2)$ freezes in the infrared and that the exchanged gluon gets an effective tachyonic mass. We verify that identical results are obtained in the Coulomb and Feynman gauges.

Keywords: Non-perturbative Effects, Confinement, QCD, Asymptotic Freedom.

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Contents

1. Introduction

In perturbative QCD (PQCD) calculations of S-matrix amplitudes quarks and gluons are assumed to form free asymptotic states at the initial and final times, $t \to \pm\infty$. It is recognized that this is at variance with observations — partons actually bind to form colour singlet hadrons which are the true asymptotic states. Consequently, the applications of perturbation theory are restricted to so-called infrared safe observables in processes characterized by a large momentum scale Q. All predictions are subject to power corrections $(\Lambda/Q)^n$, where $\Lambda \sim 200\,\mathrm{MeV}$ is the fundamental QCD scale.

It has been noted [1] that PQCD predictions can nevertheless be successfully extrapolated to low scales $Q \sim \Lambda$, assuming that the Q-dependence of the running coupling $\alpha_s(Q^2)$ "freezes" at a hadronic scale of order Λ. Confinement appears to change momentum distributions only in a mild way, with PQCD distributions of partons being reflected in those observed for hadrons. This motivates us to study whether PQCD can be modified so that its use can be extended to low Q^2 without having to introduce the freezing effects "by hand".

Formally, there is considerable freedom in making a perturbative expansion. The standard arguments justifying an expansion, namely

- The initial and final times are taken to infinity along a ray slightly tilted *wrt.* the real axis, and

- The asymptotic configurations have a non-vanishing overlap with the true ground state of the theory

allow many choices of *in*- and *out*-states. The existence of an overlap with the true ground state is in practice an assumption, even in the case of standard PQCD where the asymptotic states are taken to be the empty "perturbative vacuum". Considering the central importance of perturbation theory in applications of field theory it seems desirable to explore the properties of expansions with different asymptotic states.

Here we will study the effect of adding gluons to the perturbative vacuum. It is natural to consider this since the true QCD ground state is believed to be a condensate of gluons. Conceivably, the background gluons may mimic the properties of the true gluon condensate sufficiently to make the perturbative expansion express some of the confinement physics already at low orders. In any case, the above formal arguments justifying such a modified perturbative expansion are as compelling as those of standard PQCD.

The specific modification of the asymptotic state we consider has been called the "Perturbative Gluon Condensate" [2]. Background gluons with energies smaller than a given scale Λ are introduced by modifying the Feynman $i\varepsilon$-prescription of the gluon propagator in the following way:

$$\begin{aligned}\frac{1}{k^2+i\varepsilon} \longrightarrow \frac{1}{(k+i\varepsilon)^2} &\equiv \frac{1}{k^2+i\varepsilon} + \frac{i\pi}{2|\mathbf{k}|}\Big[\delta(k_0-|\mathbf{k}|)+\delta(k_0+|\mathbf{k}|)\Big]\Theta(\Lambda-|\mathbf{k}|) \\ &= \frac{\Theta(|\mathbf{k}|-\Lambda)}{k^2+i\varepsilon} + \frac{1}{2}\left[\frac{1}{(k_0-i\varepsilon)^2-\mathbf{k}^2}+\frac{1}{(k_0+i\varepsilon)^2-\mathbf{k}^2}\right]\times \\ &\quad \times\Theta(\Lambda-|\mathbf{k}|)\,, \end{aligned} \tag{1.1}$$

where $1/(k^2+i\varepsilon)$ denotes the ordinary Feynman $i\varepsilon$-prescription and $1/(k+i\varepsilon)^2$ denotes the modified one. As was shown [2] for scalar fields, a perturbative calculation of any Green function G using the modified propagator (1.1) is equivalent to a superposition of standard calculations using Feynman propagators with gluons added to the asymptotic states, schematically

$$\langle 0|\, G\, |0\rangle \longrightarrow \left(\prod_{|\mathbf{k}|<\Lambda}\sum_{n_\mathbf{k}=0}^{\infty} c_{n_\mathbf{k}}\right)\left\langle \prod_{\mathbf{k}}(g_\mathbf{k})^{n_\mathbf{k}}\right|\, G\, \left|\prod_{\mathbf{k}}(g_\mathbf{k})^{n_\mathbf{k}}\right\rangle . \tag{1.2}$$

Here the $n_\mathbf{k}=0$ term corresponds to the unmodified expansion, the $c_{n_\mathbf{k}}$ are known constants and the sum is over on-shell gluons $g_\mathbf{k}$ of momentum $\mathbf{k}$ and energy $|\mathbf{k}|<\Lambda$. We will show here that gauge invariance is maintained when both gluon and ghost propagators are modified according to eq. (1.1).

Physically, the modified asymptotic states imply scattering off the "background" gluons which prevents the creation of gluons with $|\mathbf{k}|<\Lambda$. Technically this can be seen from the sign change (1.1) of $i\varepsilon$ in the free gluon propagator which removes pinches between positive and negative energy poles in loop integrals. For a fermion propagator, such a change of $i\varepsilon$ would be equivalent to filling all fermion (or antifermion) levels up to a Fermi momentum Λ, and consequently preventing fermion pair production in accordance with the Pauli exclusion principle. We are motivated to study the analogous modification of the gluon propagator as a way of avoiding the production of soft gluons in perturbation theory. Since we effectively superpose calculations with different numbers of background gluons as indicated in eq. (1.2), we need not specify the wave function of such a "Dirac gluon sea" (cf. [3]). We shall refer to the physics based on the modified gluon propagator (1.1), with the standard Feynman $i\varepsilon$-prescription for quark propagators, as "Perturbative Gluon Condensate Dynamics", or PGCD. Formally, the PGCD expansion appears as justified as ordinary PQCD.

The introduction of a fixed momentum scale Λ in the PGCD propagator (1.1) seems to break Lorentz invariance. The perturbative expansion of the amplitude for a given process will depend on the reference frame, since the scale Λ is frame independent. Formally the series sums to the same (Lorentz covariant) result in any frame, but the rate of convergence is frame dependent. The situation is in this sense analogous to the well-known freedom of choice in the renormalization scale. Physical arguments must be used to choose an optimal frame for each process. This is in fact commonly done in hadron phenomenology. The non-relativistic quark model describes hadrons in their rest frames, whereas the parton model is formulated in the infinite momentum frame.

We should emphasize that the boost properties of bounds states are in general extremely complicated [4]. In QED, positronium wave functions and energy levels are nearly always evaluated in the rest frame, and most efficiently using non-covariant methods such as NRQED [5]. Not even general features such as the Lorentz contraction of QED bound states have (to our knowledge) been explicitly demonstrated in perturbation theory. In QCD we face the extra challenge that the gluon condensate ground state is boost invariant: the gluons carry momenta of $\mathcal{O}(\Lambda_{\rm QCD})$ in any frame. This feature can clearly not be described using perturbation theory — the best we can do is to approximate the true ground state with background gluons whose momenta are the same in any frame, as in eq. (1.1).

In this paper we consider the effects of PGCD on the static quark potential. This implies an automatic frame choice since the static potential is defined only in the "rest frame" of the static sources. We shall not further discuss the important and non-trivial question of Lorentz invariance. The question of frame choice for a general process is beyond the scope of this paper.

According to the Kinoshita-Lee-Nauenberg (KLN) theorem [6] all infrared singularites cancel if one sums over incoming and outgoing states that are degenerate in energy. Our procedure of adding soft gluons to the *in*- and *out*-states introduces a similar smearing of the physical observables. It has in fact been shown [7, 8, 9] that the "KLN-cancellations" can be accounted for using a similar modification of the $i\varepsilon$-prescription as the one we study here. As discussed in [9] the effects of the KLN-cancellation can be thought of as a "KLN vaccum" and the non-vanishing interactions with the vacuum as "perturbative condensates". Thus the physical picture appears similar to the PGCD. The KLN-cancellations are valid in any field theory irrespective of whether there is confinement or not, and the energy-resolution (corresponding to Λ) can be arbitrarily small. In our interpretation the scale of soft gluons is a physical feature related to the ground state of QCD.

The purpose of this paper is two-fold. On the one hand we want to investigate whether the PGCD boundary conditions give a perturbative expansion which captures some of the physics of QCD at long distances, while leaving unchanged standard perturbative results at short distances. As a first test case we calculate the QCD potential between two static colour sources in a colour singlet state [10]. We compare the ultraviolet and infrared properties of the static PGCD potential with results obtained using ordinary PQCD. The second purpose of this paper is to check explicitly that the perturbative gluon condensate framework is gauge invariant. Hence we do the calculation both in a physical and in a covariant gauge, namely the Coulomb and Feynman gauges.

2. Calculation of static potential

The QCD potential $V(Q^2)$ between two static colour sources can be defined in a gauge invariant way from a Wilson loop [10]. At lowest order the potential is just given by one-gluon exchange, $V(Q^2) = -C_F g^2/Q^2$, where g^2 is the strong coupling and $q^2 = -Q^2 = -\mathbf{q}^2$ is the squared momentum transfer which is purely space-like in the static approximation, i.e. $q_0 = 0$. The PGCD $i\varepsilon$-prescription does not change this lowest order result since the coupling of the background gluons to a source with large mass M is suppressed by Λ/M. At higher orders the fixed coupling g^2 is replaced by the running coupling after renormalization. Including all higher-order corrections in the running coupling gives an effective charge $\alpha_V(Q^2)$ defined by

$$V(Q^2) \equiv -4\pi C_F \frac{\alpha_V(Q^2)}{Q^2}\,, \tag{2.1}$$

where $C_F = (N_C^2-1)/2N_C = 4/3$ for QCD. In the following we will calculate $\alpha_V(Q^2)$ to one-loop order using the PGCD $i\varepsilon$-prescription (1.1). For convenience we define the one-loop correction $\hat{\Pi}(Q, Q_0, \Lambda)$ so that the leading order result is factored out,

$$\alpha_V(Q^2) = \alpha_V(Q_0^2)\left[1 + \hat{\Pi}(Q, Q_0, \Lambda) + \cdots\right], \tag{2.2}$$

where Q_0 is the renormalization point, i.e. $\hat{\Pi}(Q_0, Q_0, \Lambda) = 0$.

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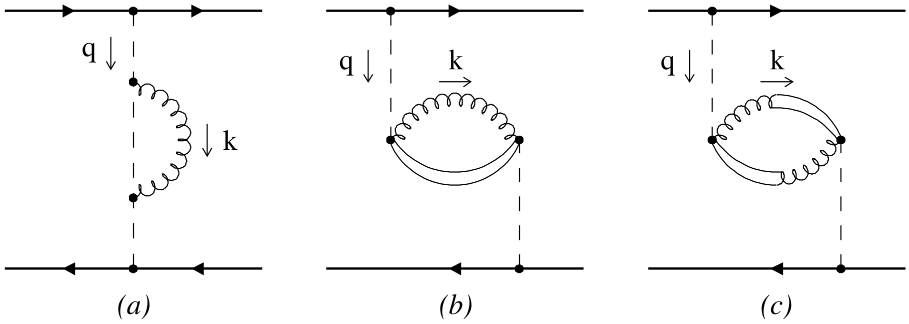

Figure 1: One-loop diagrams contributing to the static potential in Coulomb gauge. The thick lines represent the static quarks, and the dashed lines the instantaneous Coulomb propagators. The curly and double lines represent the A and E-field propagators, respectively. There is also a mixed A and E-field propagator which appears in (c).

2.1 Coulomb gauge

Coulomb gauge is the most natural gauge for calculating the static potential [11], although the Feynman rules are not as simple as in a covariant gauge such as Feynman gauge. Here we will use the Feynman rules of Coulomb gauge given by Feinberg [12]. The diagrams contributing to the static potential at one-loop order in Coulomb gauge are shown in figure 1. For clarity we do not include the contribution from light quarks, which is the same as in standard PQCD.

Using dimensional regularization the contribution to the unrenormalized one-loop correction Π from the Coulomb self-energy diagram shown in figure 1a is

$$\Pi_a = 3ig^2 C_A \mu^{4-n} \int \frac{d^n k}{(2\pi)^4} \mathbf{q}_i \mathbf{q}_j \left(\delta_{ij} - \frac{\mathbf{k}_i \mathbf{k}_j}{\mathbf{k}^2} \right) \frac{1}{\mathbf{q}^2 (\mathbf{q}-\mathbf{k})^2} \frac{1}{(k+i\varepsilon)^2} , \tag{2.3}$$

where $C_A = N_C = 3$, n is the number of dimensions ($n < 4$), μ is the arbitrary dimensional regularization scale, the subscripts i, j denote the space components ($i, j = 1, 2, 3$), and the $i\varepsilon$-prescription for the transverse gluon propagator is given in eq. (1.1). We have written the integrand in 4 dimensions since we will not be interested in constant contributions to Π. This corresponds to a specific choice of renormalization scheme. The k_0-integral is conveniently done in Minkowski space using ordinary residue calculus, and vanishes for $|\mathbf{k}| < \Lambda$ since the poles at $k_0 = \pm|\mathbf{k}|$ are then on the same side of the real axis. The result is symmetric under $\mathbf{k} \leftrightarrow \mathbf{q} - \mathbf{k}$ and can be expressed as

$$\begin{aligned} \Pi_a = 3g^2 C_A \mu^{4-n} \int \frac{d^{n-1} k}{(2\pi)^3} \left(1 - \frac{(\mathbf{k} \cdot \mathbf{q})^2}{\mathbf{k}^2 \mathbf{q}^2} \right) \times \\ \times \frac{1}{(\mathbf{q}-\mathbf{k})^2} \frac{1}{2} \left[\frac{\Theta(|\mathbf{k}| - \Lambda)}{2|\mathbf{k}|} + \frac{\Theta(|\mathbf{q}-\mathbf{k}| - \Lambda)}{2|\mathbf{q}-\mathbf{k}|} \right] , \end{aligned} \tag{2.4}$$

where the Θ-functions reflect the modified $i\varepsilon$-prescription.

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To simplify the remaining integrations it is convenient to choose

$$x = \frac{|\mathbf{k}|}{Q}\,, \qquad y = \frac{|\mathbf{q}-\mathbf{k}|}{Q} \tag{2.5}$$

as new integration variables, with measure

$$\int d^{n-1}k = \int_0^\infty dx \int_{|x-1|}^{x+1} dy \int_0^{2\pi} d\varphi\, Q^{n-1} x^{n-3} y\,, \tag{2.6}$$

where we have again dropped terms proportional to $(n-4)$ in the angular integral. The integration over the azimuthal angle φ gives 2π and the remaining integral becomes

$$\begin{aligned}\Pi_a = 3C_A \frac{g^2}{4\pi^2}\left(\frac{\mu}{Q}\right)^{4-n} \int_0^\infty dx\, x^{n-4} \int_{|x-1|}^{x+1} dy \frac{-x^4-y^4+2x^2y^2+2x^2+2y^2-1}{16x^2y^2} \times \\ \times\, [y\Theta(x-\lambda)+x\Theta(y-\lambda)]\,,\end{aligned} \tag{2.7}$$

where $\lambda = \Lambda/Q$. Before evaluating the integral we shall add the contributions from the remaining diagrams to the integrand.

According to the rules given by Feinberg [12], the contribution to Π from the sum of the vacuum-polarization diagrams in figure 1b and c is

$$\begin{aligned}\Pi_{b+c} = ig^2 C_A \mu^{4-n} \int \frac{d^n k}{(2\pi)^4}\left(\delta_{ij} - \frac{(\mathbf{q}-\mathbf{k})_i(\mathbf{q}-\mathbf{k})_j}{(\mathbf{q}-\mathbf{k})^2}\right)\left(\delta_{ij} - \frac{\mathbf{k}_i\mathbf{k}_j}{\mathbf{k}^2}\right) \times \\ \times\, \frac{k_0^2 + \frac{1}{2}[\mathbf{k}^2 + (\mathbf{q}-\mathbf{k})^2]}{\mathbf{q}^2 (k+i\varepsilon)^2 (q-k+i\varepsilon)^2}\,.\end{aligned} \tag{2.8}$$

After integrating over k_0 and φ and using (2.5) and (2.6) this becomes

$$\begin{aligned}\Pi_{b+c} = C_A \frac{g^2}{4\pi^2}\left(\frac{\mu}{Q}\right)^{4-n} \int_0^\infty dx\, x^{n-4} \int_{|x-1|}^{x+1} dy \frac{x^4+y^4+6x^2y^2-2x^2-2y^2+1}{16x^2y^2} \times \\ \times \left[y\frac{3x^2+y^2}{x^2-y^2}\Theta(x-\lambda) + x\frac{3y^2+x^2}{y^2-x^2}\Theta(y-\lambda)\right].\end{aligned} \tag{2.9}$$

Adding the Coulomb self-energy and vacuum-polarization contributions of eqs. (2.7) and (2.9) gives

$$\begin{aligned}\Pi = C_A \frac{g^2}{4\pi^2}\left(\frac{\mu}{Q}\right)^{4-n} \int_0^\infty dx\, x^{n-4} \times \\ \times \int_{|x-1|}^{x+1} dy \left[\frac{7x^4+y^4-2x^2-2y^2+1}{4x^2(x^2-y^2)} y\Theta(x-\lambda) + \right. \\ \left. + \frac{7y^4+x^4-2y^2-2x^2+1}{4y^2(y^2-x^2)} x\Theta(y-\lambda)\right],\end{aligned} \tag{2.10}$$

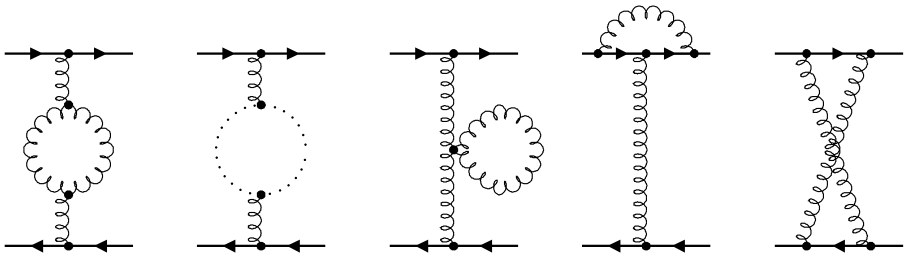

Figure 2: One-loop diagrams contributing to the static potential in Feynman gauge. The thick lines represent static quark propagators, the curly lines gluon propagators and the dotted lines ghost propagators. For clarity the vertices have been marked with dots.

Note that the apparent pole at $x = y$ cancels between the two terms in the integrand. Doing the integrals we find the result for the unrenormalized one-loop correction to the static potential,

$$\Pi(Q, \mu, \Lambda) = C_A \frac{g^2}{4\pi^2} \left[\frac{11}{6} \ln \frac{2\mu}{Q(2\lambda + 1)} + \frac{11}{6} \frac{1}{4 - n} + \frac{4}{3}\lambda^2 + C + \right. \\ \left. + \frac{(2\lambda - 1)(4\lambda^3 + 2\lambda^2 - 5\lambda + 3)}{12\lambda} \ln \frac{2\lambda + 1}{|2\lambda - 1|} \right], \quad (2.11)$$

where $\lambda = \Lambda/Q$ and C is a renormalization-scheme-dependent constant. This is the main result of our calculation. Before analysing it in more detail we check that we get the same result if we do the calculation in Feynman gauge. This will at the same time constitute a non-trivial verification of the gauge invariance of the PGCD $i\varepsilon$-prescription.

2.2 Feynman gauge

The diagrams which contribute to the static potential in Feynman gauge at one-loop order are shown in figure 2. In addition to the gluon propagator corrections of figure 2a–c there is also the vertex correction of figure 2d, which has a non-abelian contribution that does not cancel against the quark wave-function renormalization, as well as the crossed box diagram of figure 2e, which has a non-abelian part that is not part of the iteration of the one-gluon exchange. In a general covariant gauge a diagram with a three-gluon vertex also contributes, but it vanishes in Feynman gauge. For more details on the diagrams that contribute in Feynman gauge and how the iteration of the one-gluon exchange works we refer to Fischler [13].

Note that we have included the diagram with a four-gluon-vertex shown in figure 2c. In dimensional regularization this diagram does not contribute to the logarithmic UV-divergence, only to a quadratic divergence which normally cancels against the other two gluon propagator corrections. However, since we are modifying the $i\varepsilon$-prescription these cancellations are no longer guaranteed and therefore we include all diagrams.

We again use dimensional regularization and calculate the integrands of all diagrams in 4 dimensions since we are not interested in constant contributions to the final expression. The result after performing the numerator and colour algebra is

$$\Pi = \frac{ig^2}{2\mathbf{q}^2} C_A \mu^{4-n} \int \frac{d^n k}{(2\pi)^4} \left[\frac{k^2 + (k+q)^2 + 4q^2 + 10k_0^2}{(k+i\varepsilon)^2 (k+q+i\varepsilon)^2} - \frac{2k_0^2}{(k+i\varepsilon)^2(k+q+i\varepsilon)^2} - \right.$$
$$- \frac{6}{(k+i\varepsilon)^2} - \frac{2q^2}{(k_0+i\varepsilon)^2(k+i\varepsilon)^2} +$$
$$\left. + \frac{q^4}{(k_0+i\varepsilon)^2(k+i\varepsilon)^2(k+q+i\varepsilon)^2} \right], \tag{2.12}$$

where each term corresponds to a specific diagram in figure 2. The $1/(k_0 + i\varepsilon)$ factor in the vertex correction and box diagrams comes from the static quark propagator and is not to be confused with the PGCD prescription (1.1). Doing the integrals over k_0 and φ and making the variable substitutions $x = |\mathbf{k}|/Q$ and $y = |\mathbf{k} + \mathbf{q}|/Q$ we are left with

$$\Pi = C_A \frac{g^2}{4\pi^2} \left(\frac{\mu}{Q}\right)^{4-n} \int_0^\infty dx\, x^{n-4} \times$$
$$\times \int_{|x-1|}^{x+1} dy \left[\frac{6x^4 + 2x^2y^2 - 3x^2 - y^2 + 1}{4x^2(x^2-y^2)} y\Theta(x-\lambda) + \right.$$
$$\left. + \frac{6y^4 + 2y^2x^2 - 3y^2 - x^2 + 1}{4y^2(y^2-x^2)} x\Theta(y-\lambda) \right]. \tag{2.13}$$

Even though the integrand is different from the one of eq. (2.10) obtained in Coulomb gauge, the final result after the integrals are done only differs from eq. (2.11) by a renormalization-scheme-dependent constant. There is thus full agreement between the two calculations.

3. Discussion of result

Our renormalized result for the one-loop contribution to the static potential using the PGCD $i\varepsilon$-prescription is

$$\hat{\Pi}(Q, Q_0, \Lambda) = C_A \frac{\alpha_V(Q_0^2)}{\pi} \left[\frac{11}{6} \ln \frac{2\Lambda + Q_0}{2\Lambda + Q} + \frac{4}{3}\frac{\Lambda^2}{Q^2} + C + \right. \tag{3.1}$$
$$\left. + \frac{2\Lambda - Q}{12\Lambda} \left(4\frac{\Lambda^3}{Q^3} + 2\frac{\Lambda^2}{Q^2} - 5\frac{\Lambda}{Q} + 3 \right) \ln \frac{2\Lambda + Q}{|2\Lambda - Q|} \right],$$

which is obtained from eq. (2.11) by making a subtraction at $Q = Q_0$. The constant C is thus determined by the condition $\hat{\Pi}(Q_0, Q_0, \Lambda) = 0$.

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A basic control of the validity of eq. (3.1) is that it agrees with the standard PQCD result in the $Q \to \infty$ limit. For $\Lambda/Q \to 0$ we get

$$\hat{\Pi}(Q, Q_0, \Lambda)\Big|_{\Lambda/Q\to 0} = C_A \frac{\alpha_V(Q_0^2)}{\pi} \left[\frac{11}{6} \ln \frac{Q_0}{Q} - \frac{\Lambda^2}{3Q^2} + \frac{\Lambda^2}{3Q_0^2} + \right. \\ \left. + \mathcal{O}\left(\frac{\Lambda^4}{Q^4} - \frac{\Lambda^4}{Q_0^4} \right) \right]. \tag{3.2}$$

Thus the ordinary asymptotic freedom [14, 15] result is retained with power-corrections Λ^2/Q^2. Returning to the complete expression (3.1) we also note that $\hat{\Pi}$ is well defined for all finite Q/Λ, including $Q = 2\Lambda$. More precisely, $\hat{\Pi}$ is continuous at $Q = 2\Lambda$ but the derivative $d\hat{\Pi}/d\ln Q$ has an (integrable) singularity at that point.

The leading power-correction in eq. (3.2) scales as Λ^2/Q^2. By contrast, in the operator product expansion one expects a Λ^4/Q^4 scaling behaviour (see [16] for a phenomenological calculation and [17] for a related discussion). In this sense our result is more similar to the gluon propagator in the manifestly gauge dependent $\langle A_\mu^2 \rangle$ gluon condensate [18], which has been argued recently to have a possible physical meaning [19, 20]. A Λ^2/Q^2 scaling of the power-corrections to the potential in momentum space was also found in an infrared renormalon analysis by Beneke [21]. In this context we note that it is not possible to make direct comparisons of results obtained for large Q with calculations made in coordinate space since the Fourier transform from momentum space to coordinate space involves an integral over all momenta Q.

The sign of the power-correction in eq. (3.2) decreases the running of the coupling since the sign of $(\Lambda^2/Q_0^2 - \Lambda^2/Q^2)$ is opposite to that of $\ln(Q_0/Q)$. An opposite behaviour, namely infrared sensitive short-distance corrections which lead to a confining potential were found recently [22]. Since this calculation was made in coordinate space the results cannot be directly compared as explained above. We also note that the infrared renormalon analysis cannot predict the sign of the power-correction, only its scaling [21]. To see whether the negative sign of the power-correction found in eq. (3.2) gives a freezing coupling or a confining potential we have to study the small Q behaviour of eq. (3.1) since a possible fixed point for the evolution equation is at $Q = 0$.

Expanding our result (3.1) in the limit $Q/\Lambda \to 0$ we find

$$\hat{\Pi}(Q, Q_0, \Lambda)\Big|_{Q/\Lambda\to 0} = C_A \frac{\alpha_V(Q_0^2)}{\pi} \left[C(Q_0, \Lambda) + 2\frac{\Lambda^2}{Q^2} + \mathcal{O}\left(\frac{Q^2}{\Lambda^2} \right) \right], \tag{3.3}$$

where $C(Q_0, \Lambda)$ is a constant. We note several interesting aspects of this. First of all we see that the only infrared-sensitive term is of the form Λ^2/Q^2; all other terms are either constant or vanish in the limit $Q/\Lambda \to 0$. Especially there is no logarithmic Q-dependence in this limit, in other words there is no logarithmic running of the coupling for small Q/Λ. (This can also easily be seen directly from eq. (3.1).) Another interesting property of (3.3) is that the sign of the quadratic infrared divergence Λ^2/Q^2 is opposite to the one found in eq. (3.2) and thus corresponds to a linear confining potential when Fourier-transformed to coordinate space.

On the other hand, the Λ^2/Q^2 term signals a possible breakdown of our expression for the static potential at small Q^2. A closer analysis of its origin in the Feynman gauge calculation shows that it arises in the diagrams with insertions in the single gluon propagator shown in figure 2a–c. Power counting shows that this is also true in a general covariant gauge. Since these insertions can be iterated the corresponding corrections should be resummed as a geometric series,

$$\begin{aligned} V(Q^2) &= -4\pi C_F \frac{\alpha_V(Q_0^2)}{Q^2}\left[1+\hat{\Pi}(Q,Q_0,\Lambda)+\cdots\right] \\ &= -4\pi C_F \frac{\alpha_V(Q_0^2)}{Q^2}\left[1+\tilde{\Pi}(Q,Q_0,\Lambda)+2C_A\frac{\alpha_V(Q_0^2)}{\pi}\frac{\Lambda^2}{Q^2}+\right. \\ &\qquad\left.+\left(2C_A\frac{\alpha_V(Q_0^2)}{\pi}\frac{\Lambda^2}{Q^2}\right)^2+\cdots\right] \\ &= -4\pi C_F \frac{\alpha_V(Q_0^2)}{Q^2-\nu^2}\left[1+\tilde{\Pi}(Q,Q_0,\Lambda)+\cdots\right], \end{aligned} \tag{3.4}$$

where $\nu^2 = 2C_A\alpha_V(Q_0^2)\Lambda^2/\pi$ is a tachyonic effective gluon mass squared, $m^2_{g,\text{eff}} = -\nu^2$ and $\tilde{\Pi}$ is the remainder of $\hat{\Pi}$ after subtracting the quadratically divergent contribution ν^2/Q^2. At higher orders in g^2 there will be other contributions $\propto \Lambda^2/Q^2$ which will make the effective mass scale dependent. We note that according to Chetyrkin, Narison and Zakharov [17] the phenomenology of a tachyonic gluon mass is quite successful and suggests $\nu^2 \sim 0.5\,\text{GeV}^2$. More generally, the tachyonic pole indicates a qualitative change with decreasing Q^2 in the physics described by PGCD. The implications of this are beyond the scope of the present paper and require further study.

The remaining one-loop correction $\tilde{\Pi}$ can be absorbed into a modified running coupling $\tilde{\alpha}_V(Q^2,\Lambda^2)$, allowing our result to be expressed as

$$V(Q^2) = -4\pi C_F \frac{\tilde{\alpha}_V(Q^2,\Lambda^2)}{Q^2-\nu^2}\,. \tag{3.5}$$

Since $\tilde{\Pi}$ goes to a constant as $Q/\Lambda \to 0$ the modified coupling $\tilde{\alpha}_V(Q^2,\Lambda^2)$ freezes in the infrared. On the other hand, at large Q/Λ, $\tilde{\Pi}$ agrees with the standard PQCD result for $\hat{\Pi}$ up to power corrections of $\mathcal{O}(\Lambda^2/Q^2)$. Thus $\tilde{\alpha}_V(Q^2,\Lambda^2)$ equals the ordinary $\alpha_V(Q^2)$ for large Q/Λ.

To see in more detail how $\tilde{\alpha}_V(Q^2,\Lambda^2)$ freezes in the infrared it is useful to consider the one-loop β-function for this coupling,

$$\frac{d\,\tilde{\alpha}_V(Q^2,\Lambda^2)}{d\ln Q} = -\tilde{\beta}_0\left(\frac{\Lambda}{Q}\right)\frac{\tilde{\alpha}_V^2(Q^2,\Lambda^2)}{\pi}+\cdots\,. \tag{3.6}$$

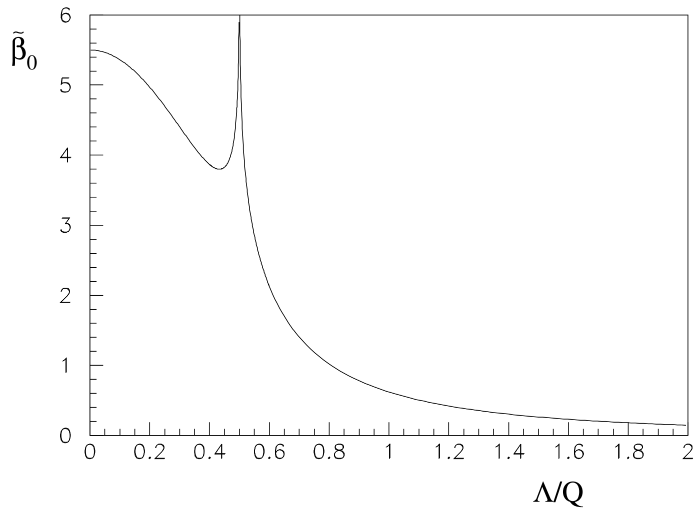

Figure 3: The one-loop coefficient $\tilde{\beta}_0(\Lambda/Q)$ of the β-function for the modified running coupling $\tilde{\alpha}_V(Q^2, \Lambda^2)$.

Taking the derivative of $\tilde{\Pi}$ with respect to $\ln Q$ we find

$$\tilde{\beta}_0\left(\frac{\Lambda}{Q}\right) = C_A\left[\frac{5}{6} - 2\frac{\Lambda^2}{Q^2} + \left(2\frac{\Lambda^3}{Q^3} - \frac{\Lambda}{Q} + \frac{1}{4}\frac{Q}{\Lambda}\right)\ln\frac{2\Lambda + Q}{|2\Lambda - Q|}\right] \tag{3.7}$$

which is plotted in figure 3. The figure shows that the running of the coupling has essentially ceased for $Q \lesssim \Lambda$. From this it follows that if ν^2 is small compared to Λ^2 then the coupling freezes in the infrared before the pole at $Q^2 = \nu^2$ is reached. The figure also illustrates the logarithmic singularity of $\tilde{\beta}_0$ at $Q = 2\Lambda$.

4. Summary and conclusions

We have explored the freedom to modify the boundary conditions of the perturbative expansion in QCD. More precisely we considered a specific modification, called Perturbative Gluon Condensate Dynamics or PGCD, where a low-energy "sea" of gluons is added to the asymptotic states by modifying the $i\varepsilon$-prescription for gluon (and ghost) propagators. As a consequence the gluon degrees of freedom freeze below a scale Λ, analogously to the behaviour of fermions in a Landau liquid. The gluon sea will scatter high-energy quarks and gluons, preventing them from forming free asymptotic states.

In order to investigate the physical relevance of the PGCD expansion we calculated the one-loop correction $\hat{\Pi}$ to the QCD potential between a static quark-antiquark pair. For large Q^2 we found that $\hat{\Pi}$ is unchanged up to power-corrections of $\mathcal{O}(\Lambda^2/Q^2)$. Thus the short distance structure of PGCD agrees with standard PQCD. At small Q^2, on the other hand, we found infrared-sensitive contributions to $\hat{\Pi}$ of $\mathcal{O}(\nu^2/Q^2)$ which after resummation give the gluon a tachyonic mass $m^2_{g,\mathrm{eff}} = -\nu^2$. The remaining part of $\hat{\Pi}$ is constant in the limit $Q/\Lambda \to 0$ and gives an effective coupling $\tilde{\alpha}_V(Q^2, \Lambda^2)$ which freezes for $Q \lesssim \Lambda$.

Our result may be summarized by the expression for the static potential

$$V(Q^2) = -4\pi C_F \frac{\tilde{\alpha}_V(Q^2, \Lambda^2)}{Q^2 - \nu^2} = -4\pi C_F \frac{\tilde{\alpha}_V}{Q^2}\left(1 + \frac{\nu^2}{Q^2} + \cdots\right). \tag{4.1}$$

By comparison we recall that at a finite *quark* density, described by modifying the $i\varepsilon$-prescription of the quark propagator, Debye screening generates a *positive* gluon mass squared. In coordinate space the ν^2/Q^2 correction term in (4.1) corresponds to a linear confining potential. The physical interpretation of our results for $Q^2 \lesssim \nu^2$ requires further study.

Our renormalized one-loop correction (3.1) to the static potential has a non-trivial dependence on Λ/Q. The fact that we obtained the same result in two quite different gauges strongly suggests that the PGCD prescription preserves QCD gauge invariance order by order in α_s. It would be desirable to prove this more generally.

Acknowledgments

We are grateful for helpful discussions with S.J. Brodsky, F. Sannino and K. Zalewski. J.R. would like to thank Nordita for its hospitality during a visit when parts of this work were done. This work was supported in part by the EU Networks “Hadron Physics with High Energy Electromagnetic Probes”, contract EBR FMRX-CT96-0008 (PH), “Electron Scattering Off Confined Partons”, contract HPRN-CT-2000-00130 (PH) and “Quantum Chromodynamics and the Deep Structure of Elementary Particles”, contract FMRX-CT98-0194 (DG 12 - MIHT) (JR).

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